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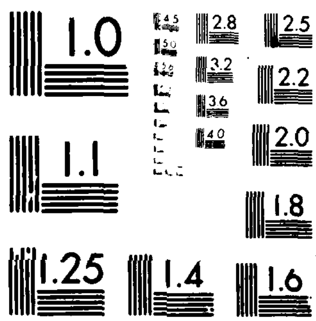

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COMPUTER ANALYSIS OF SHELLS & COMPOSITES

by

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and

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LMSC-D684018

for

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HYGROTHERMAL EFFECTS	SHELLS	COMPUTER ELASTIC-PLASTIC
COMPOSITE BONDED JOINTS	BUCKLING	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Task I Hygrothermal, viscoelastic plane stress and generalized plane strain analysis codes were developed and employed to determine the influence of hygrothermal history on laminate inplane and free edge stresses and dimensional stability. Task II Monographs were written on plastic buckling, stability and optimization of panels, and buckling of shells. These surveys include many examples of nonlinear collapse and bifurcation buckling in which moderately large deflections, nonlinear material effects, and imperfections are accounted for. Their purpose is to give the reader a physical "feel" for thin shell behavior

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that will help him to design appropriate tests and perform efficient numerical analyses with existing computer programs for the treatment of structures composed of thin sections.

FOREWARD

This final report is prepared for the Air Force Office of Scientific Research and completes the activity under the AFOSR contract F49620-77-C-0122. The research was conducted under the technical monitorship of Col. Joseph Morgan. The support of this research by AFOSR is gratefully acknowledged and appreciated.

1.0 INTRODUCTION

The purpose of this effort was to further our understanding of strong, lightweight materials and structures of interest to the Air Force. Under Task 1 methods were developed to assess the moisture and temperature sensitivity of organic matrix composites. Moisture and temperature strongly affect the load-carrying capability of structures fabricated from these materials. Under Task II certain guidelines were set forth for engineers and designers of lightweight structures fabricated from composites as well as other materials. This report summarizes the progress on these tasks.

2.0 STATEMENT OF WORK

The work statement for the two tasks in this effort is summarized below:

2.1 Task I - Hygrothermal Effects in Composite Laminates

2.1.1 Code Development

a. An existing finite element code which analyzes temperature (or moisture concentration) profiles under transient or steady state conditions will be coupled to the mechanical analyzer finite element program described immediately below.

b. The existing procedures employed to analyze viscoelasticity in laminates described by classical laminated plate theory will be coupled with a generalized plane strain finite element program which currently analyzes the influence of free edges on the thermoelastic response of the laminate.

c. The influence of moisture in the model analysis will include both physical swelling (i.e., a moisture expansion coefficient) and a shift in viscoelastic properties equivalent to the time-temperature shift factor. These procedures to account for moisture dependent alteration of mechanical properties will be implemented in both the laminate and finite element codes.

2.1.2 Analysis of Hygrothermal Effects

a. The effect of constant hygrothermal exposure on the alteration of internal residual stresses caused by differences in moisture and thermal expansion coefficients and moisture altered viscoelastic response will be analyzed by laminate and finite element codes.

b. Laminate free edge stresses as altered by constant and cyclic hygrothermal environments will be determined by finite element modeling.

c. The combined effects of hygrothermally altered free edge and internal stresses on the macromechanical response of selected laminates will be studied by finite element modeling.

2.2 Task II - Computerized Analysis of Plates & Shells

Develop a reference guide for practicing engineers who have to solve shell problems. The guide will include a review of computerized analysis and modeling of shell and panel structures with emphasis on stability. It will serve as a useful companion to computer programs for shell analysis by helping the engineer to set up appropriate analytical models.

3.0 SUMMARY OF RESEARCH RESULTS

This section presents a summary of results obtained under this contract. For each task, an overall summary is presented which references technical publications that contain details of the technical work. In Section 4, each of these publications is listed together with a brief summary of its content.

3.1 Task I - Hygrothermal Effects in Composite Laminates

Interpretation of analyses of free edge stresses or of inplane stresses determined by plane stress lamintion theory depends critically on determining the range of temperature and moisture where time-independent elastic analysis is appropriate and the range of temperature, moisture and mechanical loading where time-dependent linear viscoelasticity can be applied (Refs. 1,2,4).* We have correlated the predicted and experimentally measured laminate response with internal residual stresses that are altered by long-term exposure to moisture at room and elevated temperatures and subsequent hygrothermal cycles (Ref. 1). Based on the excellent correlation using the plane-stress laminate code, a linear viscoelastic module was developed for calculating the time, temperature and moisture dependences of all six components of stress or strain. This module was incorporated into a finite element code that analyzes nonlinear behavior by the initial stress method. The viscoelastic module described in the accompanying report has been combined with LMSC's FREE*EDGE, generalized plane strain, finite element code. The viscoelastic free edge stress analysis of a single lap bonded joint under hygrothermal and mechanical loading is described in Ref. 3.

3.2 Task II - Computerized Analysis of Plates & Shells

Volumes were written on plastic buckling of shells, interactive computer programs for optimum design of stiffened composite cylindrical panels, equations governing the analysis of shells and hybrid bodies (shells and solid regions combined), and buckling of shells and panels. The emphasis in all of the work, summarized in the next section, is on explanation of the physics of the behavior of shell and shell-like structures.

Reference 5 is an extensive survey of recent developments in the field of plastic buckling; Ref. 6 presents two interactive computer programs for weight optimization of stiffened composite cylindrical panels with general and local instability constraints; Ref. 7 contains equations governing stress, stability, and vibration of complex shells of revolution, hybrid bodies of revolution, and general shells; and Ref. 8 surveys the field of shell buckling, including nonlinear collapse, bifurcation buckling, and imperfection sensitivity.

4.0 REPORTS AND PUBLICATIONS

This section contains a list of technical reports and publications that have been prepared in connection with the research conducted under this contract. A summary of the content of each of these publications is included. Taken together, these publications constitute a detailed record of all of the results from the contract.

4.1 Task I - Hygrothermal Effects in Composite Laminates

1. "Viscoelastic Analysis of Hygrothermally Altered Laminate Stresses and Dimensions" by F. W. Crossman and D. L. Flaggs, LMSC Report D633086, November 1978

   In polymer matrix composites, absorbed moisture alters the composite viscoelastic response by causing a swelling and plasticization of the matrix. In its action as a swelling agent, a given concentration of evenly distributed absorbed moisture results in a volumetric expansion of the epoxy matrix in a manner analogous to the expansion caused by an increase in temperature. Moisture induced swelling can be accounted for in the standard thermoelastic laminate analysis by determining the appropriate moisture expansion coefficients for the unidirectional lamina and coupling the mechanical analysis to a computation of the through-thickness moisture concentration profile, which is found from the solution of the equations describing volumetric diffusion.

The second effect of absorbed moisture--plasticization of the matrix--must also be accounted for in the analysis of laminate response. In this report the single integral theory of linear viscoelasticity pioneered by Shapery was employed to analyze the effect of matrix plasticization. This model is useful because it requires a minimal number of experiments to predict response under complex loading/exposure histories. The method hinges on determining a master relaxation modulus vs. time curve and the corresponding time-temperature-moisture shift factors which bring data generated at any arbitrary temperature and absorbs moisture level into coincidence with the master curve.

Under this contract a one-dimensional analysis of moisture diffusion through the thickness of a laminate was coupled with the viscoelastic laminate code developed previously at LMSC. The ability to describe the shift factor as a function of both temperature and moisture was included in the code. In this form the viscoelastic laminate code is capable of calculating ply-by-ply laminate response to any loading history described by incremental changes in time, temperature, humidity, or mechanical loads.

The plane stress laminate code was then employed to predict the out-of-plane warping of $[0_4/90_4]_T$ laminates exposed to temperature and humidity. Significant changes occurred in warping during exposure to constant humidity/temperature and subsequent hygrothermal cycling. These calculations were compared to experimental data generated previously at LMSC.

The excellent correlation demonstrated the utility of the viscoelastic model for predicting long term hygrothermal effects based on data developed in short-time stress relaxation tests.

2. "Dimensional Stability of Composite Laminates During Environmental Exposure" by F. W. Crossman and D. L. Flaggs, SAMPE Journal, July/August 1979, pp 15-20

A computational procedure based on a linear viscoelastic description of laminated composite plates is applied in this paper to analyze dimensional stability. The dimensional stability is found to be sensitive to time, temperature moisture and radiation exposure.

Experimental verification of the linear viscoelastic numerical analysis was obtained from a comparison of predicted and measured warping of $[0_4/90_4]_T$ CY70/339 laminates during long term (up to one year) exposure to constant temperature and humidity environments.

Dimensional charges were then calculated for symmetric laminates for space structures that were designed to exhibit a nearly zero coefficient of thermal expansion ($.018 \times 10^{-6}/°F$). Inplane strain changes ranged from 10 to 40 $\mu\varepsilon$ during exposure to hygrothermal histories simulating air conditioned storage prior to launch and description of water during space orbiting.

Equations are also presented that allow the effect of space radiation exposure on composite dimensional stability to be interpreted in terms of alteration of viscoelastic material constants.

3. "Viscoelastic Response of a Bonded Joint Due to Transient Hygrothermal Exposure" by D. L. Flaggs and F. W. Crossman, presented at the Winter Annual Meeting of ASME, December 1979 and published in the volume "Modern Developments in Composite Materials and Structures", ed J. R. Vinson, ASME, New York, 1979, pp 299-314

In this paper the procedures developed for plane stress viscoelastic analysis of moisture, temperature, and time dependent laminate deformation described in Reference 1 were incorporated in a generalized plane strain analysis of free edge stresses in laminates and bonded joints. Incremental changes in the two-dimensional moisture concentration fields were calculated by the LMSC code SORPTION*ANALYSIS. The linear elastic LMSC computer code FREE*EDGE was modified under this contract to include a viscoelastic stress/strain module. The resulting code FREE*VIS could, with the output from SORPTION*ANALYSIS, determine incremental changes in both inplane and through-thickness stress components during hygrothermal exposure.

To illustrate the use of this computational procedure the time-temperature/moisture response of a single-lap shear joint was investigated by assuming a thermorheologically simple viscoelastic material description for the polymeric adhesive in conjunction with the three-dimensional generalized plane strain finite element analysis. Using this method of analysis, a realistic stress-free temperature of 201.8°F was calculated for the bonded joint, assuming an exponential cooling profile from the 250°F cure temperature to room temperature. Results of transient visco-elastic and elastic analyses are presented for bonded joints with 2024-T3 aluminum adherends and with T300/5208 graphite/epoxy adherends exposed for one year to a 100°F/70T RH environment. The analysis indicates the existence of a complex stress state within the adhesive joint due to simultaneous transient moisture diffusion and viscoelastic stress relaxation. A comparison of elastic and viscoelastic results for a one-month exposure period shows significant viscoelastic stress relaxation in those regions near the edge of the adhesive to which moisture had penetrated. This relaxation is due to the enhanced viscoelastic behavior caused by moisture-induced plasticization of the polymeric adhesive. It was found that stress relaxation reduces the peak shear stress near the free edge predicted by elastic analyses of single-lap shear joints, resulting in a nearly uniform shear stress distribution similar to that found in double-lap joints.

4. "Viscoelastic Analysis of Composite Laminate Response to Hygrothermal Exposure", by D. L. Flaggs and F. W. Crossman, manuscript in preparation for submission to J. Composite Materials, January 1980

This partially completed manuscript provides the details of the hygrothermal viscoelastic analysis of laminated plates that was described in Reference 1. As an extension of that report, a comparison is made between the viscoelastic and elastic inplane stresses calculated in a

$[0/45/-45/90]_s$ quasi-isotropic graphite-epoxy laminate during absorption and desorption of moisture. Large differences are found.

The response of $[\pm 45]_s$ and $[0/\pm 45/90]_s$ laminates to constant strain rate tensile loading at elevated temperatures and moisture content is illustrated. Although the fiber dominated, quasi-isotropic laminate shows a quasi-elastic behavior during short term tensile testing, large internal residual stress changes result due to viscoelastic deformation during long term exposure to moisture or high temperature.

The influence of the ratio of applied stress to the level of internal residual stress in determining the stress relaxation behavior of a crossplied laminate is examined. In some instances, the calculations demonstrate the need for holding specimens at temperature under zero applied load prior to stress relaxation or creep testing in order to generate viscoelastic data which is hot altered by simultaneous relaxation of internal laminate residual stresses.

5. PLASTIC BUCKLING, by D. Bushnell, LMSC-D673763, April 1979

The phenomenon of plastic buckling is first illustrated by the behavior of a fairly thick cylindrical shell, which under axial compression deforms at first axisymmetrically and later nonaxisymmetrically. Thus, plastic buckling encompasses two modes of behavior — nonlinear limit load collapse and bifurcation buckling. Accurate prediction of critical loads corresponding to either mode in the plastic range requires a simultaneous accounting for moderately large deflections and nonlinear, irreversible, path-dependent material behavior. A survey is given of plastic buckling which spans three areas: asymptotic analysis of postbifurcation behavior of perfect and imperfect simple structures, general nonlinear analysis of arbitrary structures, and nonlinear analysis for limit load collapse and bifurcation buckling of shells and bodies of revolution. A discussion is included of certain conceptual difficulties encountered in plastic buckling models, in particular those having to do with material loading rate at bifurcation and with the apparent paradox that use of deformation theory often leads to better agreement with tests on structures with very simple prebuckling equilibrium states than does use of the more rigorous incremental flow theory. In the survey of general nonlinear structural analysis, emphasis is given to formulation of the basic equations; various elastic-plastic material models; and strategies for solving the nonlinear equations incrementally. In the section on buckling of axisymmetric structures, numerous examples including comparisons of test and theory reveal that critical loads are not particularly sensitive to initial imperfections when the material is stressed beyond the proportional limit. A final summary includes suggestions for future work.

Most of this volume will appear as a chapter in the Pressure and Piping Division of the ASME's volume, Decade of Progress, to be published by the ASME's PVP in late 1980 or early 1981.

6. INTERACTIVE PROGRAM FOR OPTIMUM DESIGN OF STIFFENED COMPOSITE CYLINDRICAL PANELS, by David Bushnell, LMSC-D676240, June 1979 and LMSC-D766309, July 1979

The design of composite panels to avoid buckling under in-plane loads has received considerable attention recently. The objective of the effort reported here has been to create two interactive computer programs, PANEL and PANEL/CONMIN, which derive designs of stiffened composite cylindrical panels under combined in-plane loads.

The PANEL Computer Program

In PANEL the design is arrived at through use of an optimality criterion; buckling in general instability and local instability occurs at the same load or at different loads which are related to each other through a program-user-defined parameter, ϕ. The loading of the stiffened panel is assumed to result in uniform membrane strain e_{ox} and e_{oy} in both skin and stiffeners. The buckling loads are calculated by use of simple assumed displacement functions. For example, general instability is assumed to occur in the familiar $w_G(x,y) = C_G \sin(n\pi y/b) \sin(m\pi x/a)$ mode in which "b" is the width and "a" is the axial length of the cylindrical panel. The stiffeners are composed of assemblies of flat plate segments the lengths of which are large compared to the widths, and the widths of which are large compared to the thicknesses. These flat plate segments are oriented either normal or parallel to the plane of the panel skin. Buckling occurs at stress levels for which the material is elastic. The cylindrical skin can be composed of multiple layers of orthotropic material. Each layer has a unique angle of orthotropy relative to the direction of the cylinder generator (x-direction). The segments of the stiffeners are monocoque and orthotropic. Local buckling of the skin implies

$$w_{skin} = C_{skin} \sin(n_{skin}\pi y/b_o) \sin(m_{skin}\pi x/a_o)$$

Two classes of local stiffener buckling are accounted for. Local stiffener buckling modes are characterized by buckling of individual stiffener segments with no translation of the junctures between these segments. Simple support conditions are imposed at segment junctures. The other class of stiffener buckling is called "rolling." There are three kinds of stiffener rolling, one in which the panel skin participates and two in which it does not. In the first the stiffener cross-section does not deform but simply rotates about its line of attachment to the skin. In the other two rolling modes the stiffener web deforms and the portion of the cross-section attached to this web translates and rotates.

The input data for PANEL include initial guesses for shell wall layer thicknesses and stiffener dimensions and spacings. The number of stiffener segments and proportions of their widths remain fixed during a case. Also, the radius of the cylinder, material properties, and proportions of skin layer thicknesses remain constant. The ratios of stiffener segment widths to stiffener spacing may be held constant or may be design parameters. The stiffener spacings, a_o and b_o, may be held constant or may be design parameters.

The output from PANEL includes thicknesses of skin and stiffener segments, stiffener spacing, and critical buckling wave numbers for the design corresponding to simultaneous buckling in the general instability mode, local skin buckling mode, and local stiffener segment buckling modes. If prebuckling shear N_{xyo} is present, the slope of the buckling nodal lines is also provided in the output list.

The program PANEL runs on the CDC "NOS" system in the interactive mode. About 1 to 2 seconds of CP time are required on the Cyber 175 for execution of a case. PANEL sets up a file for input to PANEL/CONMIN.

The PANEL/CONMIN Computer Program

In the second of the two interactive computer programs, PANEL/CONMIN, a modified version of the computer program "PANEL" described above (now a structural analyzing subroutine called BUCKLE) is used in connection with an optimizer "CONMIN" written by Vanderplaats to obtain minimum weight designs with general and local instability constraints. A panel design obtained with use of the program PANEL by itself represents a starting point in the PANEL/CONMIN runstream. In an interactive execution of PANEL/CONMIN the user is first asked to indicate which of the design parameters are linked to decision variables. The optimization (weight minimization) then proceeds iteratively, with the user being asked at intervals whether or not he wishes to proceed with more iterations, indicate new decision variables, or terminate the run. About 5 to 30 CP seconds are required for execution of a case.

A summary of this research was presented at the Air Force-sponsored "Mechanics of Composites Review," Bergamo Center, Dayton, Ohio, 30 Oct - 1 Nov., 1979.

7. <u>COMPUTERIZED ANALYSIS OF SHELLS, VOL. 2: GOVERNING EQUATIONS</u>, by D. Bushnell, LMSC-D681421, Dec. 1979

The volume opens with a general discussion of terms in an energy functional which might be the basis from which equations governing stress, stability, and vibration analyses are derived. The energy expression includes strain energy of the shell and discrete stiffeners, kinetic energy of the shell and stiffeners, constraint conditions with Lagrange multipliers, and other terms arising from the change in direction of applied loads during deformation. Brief discussions are included of the coupling effect between bending and extensional energy needed for the analysis of layered composite shells or elastic-plastic shells, nonlinear terms, and the form that the energy expression takes upon discretization of the structure.

A chapter follows in which the energy formulation for stress, stability, and vibration analyses of an elastic curved beam is given, including thermal effects, moderately large rotations, boundary conditions, and distributed and concentrated loads. The matrix notation and type of discretization are introduced here which will later be used for the analysis of shells of revolution. Terms in the local element stiffness, mass, and load-geometric matrices are derived in terms of nodal point displacements, and it is shown how these local matrices are assembled into global matrices. The purpose of the chapter is to demonstrate the procedure for derivation of the analogous equations and quantities for shells of revolution or more complex structures.

The next chapter is on elastic shells of revolution. It opens with a summary of computer programs that exist for stress, buckling, and stability analyses of such structures. The assumptions on which these programs are based are listed and the various components of the energy functional, such as strain energy of the shell and discrete rings, are identified and derived in terms of nodal point displacements. Included are a derivation of the constitutive law for anisotropic shell walls and a formulation of nonlinear constraint conditions, which are required for the treatment of segmented or branched shells with meridional discontinuities between segments or branches. Derivations of terms in the global stiffness and load-geometric matrices and the force vector are given, with tables tracing the origin of each term. The computational strategy for calculation of critical bifurcation buckling loads in the presence of prebuckling nonlinearities is given, with an example of buckling under axial compression of a very thin cylinder. This is a simple problem to formulate but a difficult one to solve numerically, owing to the existence of closely spaced eigenvalues corresponding to nonsymmetric buckling at loads close to the load corresponding to nonlinear axisymmetric collapse. A description of various pitfalls encountered in the search for the lowest bifurcation buckling load is given, including estimates of the critical number of circumferential waves in the buckling mode. Computerized formulations and run times are compared for various discretization methods, including finite difference energy models and standard finite element models, with an example showing comparisons of rate of convergence with increasing nodal point density and computer times required to form stiffness matrices.

Hybrid bodies of revolution are discussed next. By "hybrid" is meant a body of revolution with both one-dimensionally and two-dimensionally discretized regions. The formulation is particularly useful for the stress, buckling, and vibration analyses of branched shells or ring-stiffened shells in which one is particularly interested in local effects within a distance equal to a shell wall thickness of a branch or ring. An appropriate strategy for the solution of nonlinear problems with simultaneous geometric nonlinearity and path-dependent material properties is described, including the development of the incremental constitutive law for the tangent stiffness method of treatment of elastic-plastic structures. The two-dimensionally discretized regions are modeled with use of 8-node isoparametric quadrilaterals of revolution. Details are presented on the formulation of constraint conditions for compatibility at junctions between rotationally symmetric shell segments (one-dimensionally discretized regions) and solid segments (two-dimensionally discretized regions).

The volume closes with a summary of linear equations for general shells. Surface coordinates, the first and second fundamental forms, and the definition of a shell are introduced, and the assumptions corresponding to Love's first approximation are identified. The differences in commonly used or referenced formulations are listed, including differences with regard to kinematic relations, expressions for total strain anywhere in the thickness of the shell wall, and expressions for stress and moment resultants. Comments are offered on which theory is the most suitable for engineering estimates.

8. COMPUTERIZED ANALYSIS OF SHELLS, VOL 4: BUCKLING, by D. Bushnell, LMSC-D 681517, Dec. 1979

A brief description of the two kinds of buckling, collapse and bifurcation, is first given. This is followed by a simple mathematical example involving a shallow truss, which displays most of the phenomena to be illustrated later with thin shells. Many examples of classical buckling of uniformly loaded cylindrical and spherical shells are then shown, with comparisons between test and theory to emphasize the sensitivity of these buckling loads to initial structural imperfections.

A major section follows in which the cause of failure is nonlinear collapse due to either large deflections or to both large deflections and nonlinear material behavior. In certain of the cases the predicted nonlinear collapse load is compared to a critical load calculated from a linearized bifurcation buckling model. Included in this section are descriptions of elastic-plastic collapse of cylindrical shells subjected to uniform axial compression or external pressure, elastic-plastic collapse of straight and curved pipes subjected to external pressure and bending, elastic collapse of shallow spherical caps under external pressure and elastic collapse of cylindrical panels and shells under combined axial compression and concentrated loads. The section closes with descriptions of collapse failure of axially compressed cylinders with cutouts, noncircular cylinders, and cylinders with local axisymmetric load path eccentricity.

The next major section gives examples of axisymmetric shells in which failure is due to bifurcation buckling. In all of the examples nonuniformity or nonlinearity of the prebuckling behavior is important. Several illustrations are provided of bifurcation buckling due to local edge effects and local hoop compression. These are followed by numerous examples in which the prebuckled state is characterized by meridional tension combined with hoop compression. Bifurcation buckling of internally pressurized torispherical shells, both in the elastic and in the plastic range of material behavior, is described in detail. The section closes with an example in which bifurcation buckling and axisymmetric collapse occur almost simultaneously.

The following section provides examples that illustrate the effects of boundary conditions and eccentric loading on bifurcation buckling of shells of revolution. The emphasis is on buckling of monocoque and stiffened cylindrical shells under uniform external pressure and axial compression. Examples are also given of inextensional buckling modes, which are associated with very low critical loads; of change in effective "boundary" condition due to development of a plastic region in the prebuckling phase; and dependence of the buckling load on small inward and outward axisymmetric imperfections of an axially compressed stringer-stiffened short cylindrical shell.

The next section is devoted to combined loading of cylindrical shells and nonsymmetric loading of shells of revolution. Interaction curves are given for monocoque cylinders under combined axial compression and internal or external pressure corresponding to various boundary conditions. Post-buckling configurations are shown for either axial compression or torsion combined with internal pressure. Interaction curves are also presented for ring or stringer-stiffened cylinders and angle-ply laminated cylinders. Examples of nonsymmetrically loaded shells of revolution include buckling of a payload shroud such as that shown in Fig. 4.5(a) due to nonsymmetric pressure, buckling of a ring-stiffened cylinder under combined bending and nonuniform heating, buckling of cylindrical and conical shells heated along narrow axial strips, and buckling of a steel containment vessel due to compressive stresses generated by vertical and horizontal components of ground acceleration during an earthquake.

The following section is on bifurcation buckling and collapse of ring-stiffened shells with emphasis given to cylindrical shells. The section begins with an illustration of the effect of discrete rings on the prebuckling state and general instability bifurcation buckling mode. Comparisons between test and theory are given for elastic buckling of machined specimens in a study in which the effect of axial restraint at the boundaries is investigated. Elastic-plastic buckling of a series of steel specimens is then described, followed by an example of a titanium shell which is predicted to fail by nonsymmetric bifurcation buckling when creep is neglected and by axisymmetric collapse when creep is included in the analysis. The effect on predicted buckling loads of initial imperfections and residual stresses due to weld shrinkage at stations where discrete rings are attached to a shell is illustrated for an ellipsoidal shell subjected to hydrostatic compression. The combined effects on failure of cold bending an initially flat sheet into a cylindrical shell and subsequently welding ring stiffeners to it are described. The section closes with a number of examples showing the importance in certain cases of treating discrete ring webs as flexible shell branches in analytical models for prediction of axisymmetric collapse and nonsymmetric bifurcation buckling.

The next section contains several illustrations of buckling of prismatic shells and panels, that is, structures that have a cross section that is constant in one of the coordinate directions. Included are descriptions of modal interaction in panels built up of thin sections and demonstrations of local cross section deformation in the bifurcation buckling modes of initially perfect panels. Predicted bifurcation buckling modes are shown for noncircular cylinders under axial compression and comparisons between test and theory are given for oval cylinders under external pressure. The section closes with brief discussions of the effect of transverse shear deformation on the buckling of composite plates and on the usefulness of the Southwell plot for prediction of instability failure.

The following section focuses on the sensitivity of predicted buckling loads to initial geometrical imperfections. The section opens with a chart of empirical knockdown factors for monocoque cylinders subjected to axial compression and a review of various types of pre- and post-buckling load deflection curves. The Koiter theory is briefly summarized and imperfection sensitivity factors for various systems are plotted. A design method for other than monocoque shells is outlined and illustrated for cylinders with combined axial compression and internal pressure. Charts are given that show typical measured imperfections in small laboratory models and a large industrial ring and stringer stiffened shell. Buckling interaction curves for a laminated cylindrical shell are plotted and compared to test results on an imperfect specimen. The section closes with several examples in which bifurcation buckling is stable and the structures carry increasing loads far into the post-buckling regime.

The last section demonstrates axisymmetric collapse and bifurcation buckling of bodies of revolution that consist of combinations of thin shell segments and solid segments to which shell theory cannot be applied with sufficient accuracy. An example is given of buckling of a hydrostatically compressed ring-stiffened cylinder in which the rings and portions of shell to which the rings are attached are modeled as solid regions with use of isoparametric quadrilaterals of revolution and the rest of the cylinder is modeled as a series of thin shell segments. Other examples include buckling of a spherical shell embedded in a softer elastic material and collapse of a complex cylinder-cone combination containing a frangible joint. The region in the immediate neighborhoods of notches in the frangible joint are modeled with use of solid elements.

The purpose of the many examples presented here is to give the reader a physical "feel" for shell buckling. With such knowledge the engineer will have an enhanced ability to foresee situations in which buckling might occur and to modify a design to avoid it. He will be able to set up more appropriate models for tests and analytical predictions, including failure due to buckling. The emphasis in this chapter is not on the development of equations for prediction of instability. For such material the reader is referred to the book by Brush and Almroth and the material in Volume 2.

Portions of this volume will be presented as the 1980 Annual Design Lecture at the 21st AIAA/ASME Structures, Structural Dynamics, and Materials Conference to be held May 12-14 in Seattle.

Section 5 SUGGESTED ADDITIONAL WORK

Work should proceed on additional volumes pertaining to Computerized Analysis of Shells. In particular, a short introductory volume should be written in which many examples of impressive shell structures are shown and the content of the other volumes is summarized.

A short volume on stress analysis of shells is needed. This volume would contain examples of discontinuity stresses with decay due to "elastic foundation" and shear lag effects, transverse shear deformation effects, and local "thick" shell behavior. Various types of nonlinear shell behavior would be discussed, including stress redistribution effects, prestress effects, load path eccentricity, and nonlinear material properties.

A somewhat longer volume on modal vibrations and dynamic response could also be written, including discussions of modal vibration with and without prestress, linear and nonlinear dynamic response, dynamic buckling, and fluid-structure interaction.

A volume on computer programs for shell analysis is needed, which would consist primarily of a survey of capabilities of structural analysis codes with use of an information retrieval system described in <u>Structural Mechanics Computer Programs,</u> edited by Pilkey, Saczalski, and Schaeffer (pp. 739-804, 1974).

Two other volumes containing information on discretization and tips on modeling are needed. The volume on discretization would cover shell elements, with comparisons of rates of convergence and required computer time to form stiffness matrices and solve associated systems of simultaneous equations. The volume on modeling techniques would cover strategies for solving nonlinear problems and for treatment of geometrical peculiarities, such as stiffeners and cutouts. Examples of complex cases would be included.

Taken together, the volumes would comprise a comprehensive treatise on the practical aspects of Structural Shell Analysis.

To summarize, the volumes in the total work would be:

Vol. 1	Introduction	(to be written)
Vol. 2	Governing Equations	(completed - LMSC-D681421)
Vol. 3	Discretization	(to be written)
Vol. 4	Buckling	(completed - LMSC-D 681517)
Vol. 5	Modal Vibration and Dynamic Response	(to be written)
Vol. 6	Stress	(to be written)
Vol. 7	Tips on Modeling	(to be written)
Vol. 8	Computer Programs for Shell Analysis	(to be written)